A General Approach to Network Analyzer Calibration

Kimmo J. Silvonen

Abstract—**A new general-purpose algorithm for network analyzer and test fixture calibration is presented. The TCX algorithm is able to handle most of the existing calibration methods including symmetrical test fixtures. Any combination of one-port or two-port standards can be used. There is a possibility of partial self-calibration, if one of the standards is a two-port network or a through connection. The algorithm is applied to get simple equations covering the TSD, LDX (LRL), LAX, and LMX methods (*X* being an unknown one-port or symmetrical two-port network). A transmission path is allowed between the ports of standard *X*. In the TSD method the delay line can be replaced with an attenuation network or with a matched load; also the "through" line can have an unknown delay and attenuation. A new method of root choice for LRL and similar methods in conjunction with test fixtures is described. The method of least-squares-fit can be applied, when redundant data are available. It gives an essential improvement of accuracy in the simulation of a symmetrical test fixture.**

I. Introduction

Different calibration methods have advantages compared with each other, if accuracy in various frequency bands and availability of standards are considered. For example:

In the TSD (through-short-delay, line-short-delay) method [1] the lengths of both lines are allowed to be unknown, but the reflection standard has to be exactly known. The difference of the line lengths must not be near a multiple of 180°.

In the LRL (line-reflect-line) method [2] an unknown reflection standard can be used, but one of the line lengths has to be known or zero (TRL [3]). As in the previous method, several lines are needed in wideband applications.

The LMR (line-match-reflect) and the LAR (line-attenuation-reflect) methods [4], [5] do not have a bandwidth limitation caused by the line lengths, but matched loads or matched attenuators are sometimes difficult to construct. The reflection standard does not have to be known.

Also the LSO (line-short-open) method [6] uses only one line length, but two different reflection standards are not always easily available. The line length should not be near a multiple of 90°. A zero-length line is not allowed.

There is a need of a general-purpose formulation capable of handling nearly all the known calibration methods with a single set of equations. Several attempts have been made in the existing literature to find a "unified" or a "generalized" formulation for network analyzer calibration. However, they all seem to be more or less restricted. In [7], [8] the possibility of self-calibration has not been accounted for. In [9], [10] not all possible combinations of calibration standards have been studied. The approach of [11] is iterative. The best general-purpose method published thus far is the one of Eul and Schiek [4], [5]. Yet, the use of transmission matrices leads to a situation, in which one-port and two-port standards have to be handled separately. In this paper a new very general calibration algorithm for test fixtures and network analyzers is presented. The algorithm has some advantages compared with the others. The new algorithm uses entirely S parameters, which have non-infinite values for all practical circuit elements including open and short circuits and calibration standards with a zero transmission.

At least two calibration standards are needed, if the fixture is symmetric. Normally the procedure relies on three or more standards. There is no upper limit for the number of standards. One of the standards should be a known two-port or a through connection. The second standard can either be a known two-port or a known reflective or nonreflective termination. One of the S parameters of the second standard is allowed to be unknown. With a nonreciprocal standard S_{12} and S_{21} of the second standard can both be calculated during the calibration (cf. unknown transmission in the TSD, LRL, and LAR methods). If the second standard has a zero transmission, no S parameters of it are allowed to be unknown. The optional third standard has to be used in conjunction with asymmetrical test fixtures and network analyzers. It can be used also with symmetrical test fixtures. If the third standard is unknown, it has to be symmetric [4]. In such case the S parameters of the third standard are calculated as a byproduct. Three exactly known one-port standards as in the short-open-match method [12] can be used as a special case.

Most of the previously known methods, as e.g., TSD, LRL, LD [13]–[15], LAR, LMR, LDN, LAN, LMN [5] and LSO can all be formulated using the same set of equations. An identification TCX, lead from the standards ("transmission type circuit—any circuit—unknown circuit") is proposed for the new formulation. To make the nomenclature more clear the names of the TSD and LRL methods could be changed to LDS and LDR respectively (cf. [5]). As an example the new algorithm can be applied to the LSO method allowing either the open or the short to be unknown. Such alternative combinations as line-

Manuscript received May 22, 1991; revised November 21, 1991.
The author is with Helsinki University of Technology, Otakaari 5 A SF-02150, Espoo, Finland.
IEEE Log Number 9106054.

0018-9480/92$03.00 © 1992 IEEE

short-network and line-open-network can also be formulated with TCX algorithm, if an exactly known reflection standard (short or open) is available. This kind of algorithm has not been published before.

With the LRL, LAR, LMR, and similar methods the problem of root choice is here solved in an alternative way, better suited for offset shorts and opens than the previously used method. The phase of the reflect standard does not have to be approximated anymore, when using two-tier calibration.

II. TCX Formulation

The eight-term error model [16] is used with the nomenclature of [17]. If necessary, the error model can be extended to the twelve term model as explained e.g., in [18] or [19]. In the following $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are S matrices of the standards and $\boldsymbol{M_A}$, $\boldsymbol{M_B}$, $\boldsymbol{M_C}$ respectively the measured S parameters using these standards (Fig. 1). $\boldsymbol{L}$ and $\boldsymbol{R}$ are the error network S matrices so that port one of $\boldsymbol{R}$ is on the right hand side. S parameters $\boldsymbol{M_D}$ are the measurements of the device-under-test $\boldsymbol{D}$. It should be mentioned that the equation set (1)–(4) is a combination of the four equations achieved for one standard with flow graph analysis or cascade matrices. These equations were originally published, although in a different form, in [7]. For known standards $\boldsymbol{A}$ and $\boldsymbol{B}$:

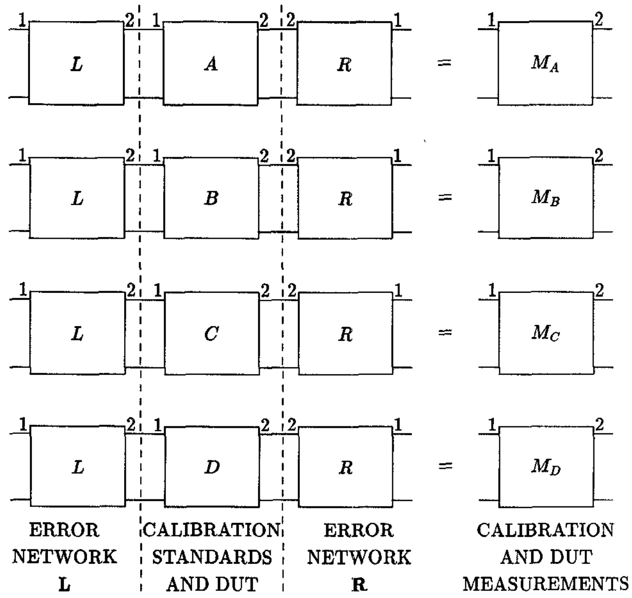

Fig. 1. Block diagram for the eight term error model.

$$M_{A21}M_{B12}A_{12}B_{21} = M_{A12}M_{B21}A_{21}B_{12}. \qquad (11)$$

The easiest way of using the equation set is to exclude (6) and (7). This allows standard $\boldsymbol{B}$ to have a zero transmission, if such standard is used. For example the LMR method does not have to be handled separately anymore,

$$\begin{bmatrix} 1 & A_{11}M_{A11} & -A_{11} & 0 & A_{21}M_{A12} & 0 \\ 0 & A_{12}M_{A11} & -A_{12} & 0 & A_{22}M_{A12} & 0 \\ 0 & A_{11}M_{A21} & 0 & 0 & A_{21}M_{A22} & -A_{21} \\ 0 & A_{12}M_{A21} & 0 & 1 & A_{22}M_{A22} & -A_{22} \\ 1 & B_{11}M_{B11} & -B_{11} & 0 & B_{21}M_{B12} & 0 \\ 0 & B_{12}M_{B11} & -B_{12} & 0 & B_{22}M_{B12} & 0 \\ 0 & B_{11}M_{B21} & 0 & 0 & B_{21}M_{B22} & -B_{21} \\ 0 & B_{12}M_{B21} & 0 & 1 & B_{22}M_{B22} & -B_{22} \end{bmatrix} \begin{bmatrix} L_{11} \\ L_{22} \\ \Delta L \\ kR_{11} \\ kR_{22} \\ k\Delta R \end{bmatrix} = \begin{bmatrix} M_{A11} \\ kM_{A12} \\ M_{A21} \\ kM_{A22} \\ M_{B11} \\ kM_{B12} \\ M_{B21} \\ kM_{B22} \end{bmatrix} \qquad \begin{matrix} (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \\ (7) \\ (8) \end{matrix}$$

where Δ means determinant (e.g., $\Delta L = L_{11}L_{22} - L_{12}L_{21}$). If more than two known standards are used, a straight forward extension of the equation set (1)–(8) is possible.

When $\boldsymbol{A}$ is fully known and one of the S parameters of standard $\boldsymbol{B}$ is unknown, the unknown parameter can be obtained from (10), [8], [17]. Should $\boldsymbol{B}$ be nonreciprocal and B_{11} and B_{22} known, (10) and (11) can be used to obtain B_{12} and B_{21}. They are the same equations as (26) and (27) in [5] written in terms of S parameters (instead of T parameters).

$$k = \frac{L_{21}}{R_{21}} \qquad (9)$$

$$(\Delta A + \Delta B - A_{11}B_{22} - B_{11}A_{22})M_{A12}M_{B21} = (\Delta M_A + \Delta M_B - M_{A11}M_{B22} - M_{B11}M_{A22})A_{12}B_{21} \qquad (10)$$

as in the formulation of Eul and Schiek [4], [5]. In fact any such combinations of standards as "Known two-port $\boldsymbol{A}$, partly known two-port or one-port $\boldsymbol{B}$, unknown symmetrical two-port or one-port $\boldsymbol{C}$," are possible with this formulation.

Equations (1)–(8) can be written in matricial form:

$$NE = G + kH \qquad (12)$$

$$G = [M_{A11}, 0, M_{A21}, 0, M_{B11}, 0, M_{B21}, 0]^T \qquad (13)$$

$$H = [0, M_{A12}, 0, M_{A22}, 0, M_{B12}, 0, M_{B22}]^T \qquad (14)$$

$$E = [L_{11}, L_{22}, \cdots, k\Delta R]^T \qquad (15)$$

$$E = N^{-1}(G + kH). \qquad (16)$$

Because $\boldsymbol{N}$, $\boldsymbol{G}$ and $\boldsymbol{H}$ are not functions of k, it is clearly seen that S parameters $\boldsymbol{E}$ vary linearly with k.

Equations (1)–(8) are solved as follows. First the unknown parameter k on the right hand side is set equal to zero and the solution

$$[x_1, x_2, x_3, x_4, x_5, x_6]^T = N^{-1}G \tag{17}$$

is found. Then we continue setting $k = 1$ (again only on the right hand side) and find a new solution

$$[z_1, z_2, z_3, z_4, z_5, z_6]^T = N^{-1}(G + H). \tag{18}$$

Coefficients $y_1 \cdots y_6$ can now be calculated

$$y_i = z_i - x_i \qquad i = 1, 2, \cdots 6 \tag{19}$$

to achieve the full solution:

$$L_{11} = x_1 + y_1 k \tag{20}$$

$$L_{22} = x_2 + y_2 k \tag{21}$$

$$\Delta L = x_3 + y_3 k \tag{22}$$

$$R_{11} = \frac{1}{k} x_4 + y_4 \tag{23}$$

$$R_{22} = \frac{1}{k} x_5 + y_5 \tag{24}$$

$$\Delta R = \frac{1}{k} x_6 + y_6. \tag{25}$$

If either k is exactly known or symmetrical error networks are assumed, the unknowns can be solved from (1)–(8) [17]. Usually k is calculated using a third measurement $\boldsymbol{M}_C$. Assuming symmetry, but not necessarily reciprocity in standard $\boldsymbol{C}$, we define:

$$\Gamma = C_{11} = C_{22} \tag{26}$$

$$T = C_{21}. \tag{27}$$

From (11) it is seen that

$$C_{12} = QC_{21} \tag{28}$$

$$Q = \frac{M_{A21} M_{C12} A_{12}}{M_{A12} M_{C21} A_{21}}. \tag{29}$$

Equations (1)–(4) are repeated for standard $\boldsymbol{C}$. By applying results (20)–(25) a set of four new equations for standard $\boldsymbol{C}$ is found:

$$\begin{bmatrix} M_{C11}x_2 - x_3 & M_{C11}y_2 - y_3 & M_{C12}x_5 & M_{C12}y_5 \\ M_{C12}x_5 & M_{C12}y_5 & (M_{C11}x_2 - x_3)Q & (M_{C11}y_2 - y_3)Q \\ M_{C21}x_2 & M_{C21}y_2 & M_{C22}x_5 - x_6 & M_{C22}y_5 - y_6 \\ M_{C22}x_5 - x_6 & M_{C22}y_5 - y_6 & M_{C21}x_2 Q & M_{C21}y_2 Q \end{bmatrix} \begin{bmatrix} \Gamma \\ k\Gamma \\ T \\ kT \end{bmatrix} = \begin{bmatrix} M_{C11} - x_1 - ky_1 \\ kM_{C12} \\ M_{C21} \\ kM_{C22} - x_4 - ky_4 \end{bmatrix}. \tag{30–33}$$

For a reciprocal two-port $\boldsymbol{C}$ $Q = 1$. If standard $\boldsymbol{C}$ is asymmetric, an extension of equation set (1)–(8) has to be used [17].

To obtain the dependent unknowns Γ, $k\Gamma$, T and kT, parameters u_i and v_i are solved in the same way as x_i and y_i:

$$\Gamma = u_1 + kv_1 \tag{34}$$

$$k\Gamma = u_2 + kv_2 \tag{35}$$

$$T = u_3 + kv_3 \tag{36}$$

$$kT = u_4 + kv_4. \tag{37}$$

If the solving of the same set of equations twice is considered a drawback, an alternative method e.g., using subdeterminants could be used. Anyhow, the calculation can be performed in real-time.

Second order equation is found for k, Γ or T using (34)–(35) and (36)–(37), respectively:

$$v_1 k^2 + (u_1 - v_2)k - u_2 = 0 \tag{38}$$

$$\Gamma^2 - (u_1 + v_2)\Gamma + u_1 v_2 - v_1 u_2 = 0 \tag{39}$$

$$v_3 k^2 + (u_3 - v_4)k - u_4 = 0 \tag{40}$$

$$T^2 - (u_3 + v_4)T + u_3 v_4 - v_3 u_4 = 0. \tag{41}$$

Only one of the equations (38)–(41) needs to be solved. In specific cases the root choice may be based on any of the unknowns: Γ, T, L_{11}, $\cdots$, ΔR or k. However, if $\boldsymbol{A}$ and $\boldsymbol{B}$ are both matched standards, T, L_{11} or R_{11} cannot be used as a basis of the root. A wrong choice tends to lead to non-physical values $|L_{22}| > 1$ and $|R_{22}| > 1$ in the test fixture calibration. It is a common practice to choose the sign based on the phase of Γ, which is normally at least approximately known. With transmission line, offset short circuit and offset open circuit standards the phase of Γ is frequency dependent. Thus, it is not very suitable for the basis of the root. In the case of two-tier calibration the best way is to utilize parameter k. With reasonably symmetrical test fixtures the complex value of $k \approx 1 + 0j$. So the other root, which is in most cases $k \approx -1 + 0j$, can easily be disregarded. In network analyzer calibration the ordinary root choice method may be better.

If $T = 0$ only two of (30)–(33) are used, namely (30) and (33). The choice of the number of equations can be controlled automatically by testing S parameters M_{C12} and M_{C21} of the measurement data or with a preliminary knowledge of the type of standard $\boldsymbol{C}$. Reflection coefficient Γ is allowed to be equal to zero only, if standard $\boldsymbol{B}$ (or $\boldsymbol{A}$ or both) is reflective. So, for example, line-short-X is a possible method even if X is a delay line, an attenuator or a matched load.

There is a free parameter α in the formulation of [4] and a corresponding free "quadrant" in [7]. The interpretation of the free parameter is the "level of non-reciprocity", which can be chosen freely. Different values of α lead to differently non-reciprocal ($L_{12} \neq L_{21}$, $R_{12} \neq R_{21}$) error networks. One of the S parameters L_{12}, L_{21}, R_{12}, R_{21} can be chosen freely, e.g.:

$$L_{12} = \alpha \neq 0 \tag{42}$$

$$L_{21} = \frac{L_{11}L_{22} - \Delta L}{L_{12}} \tag{43}$$

$$R_{21} = \frac{L_{21}}{k} \tag{44}$$

$$R_{12} = \frac{R_{11}R_{22} - \Delta R}{R_{21}}. \tag{45}$$

The values of the quantities L_{11}, L_{22}, ΔL, R_{11}, R_{22} and ΔR still remain unchanged. Thus (1)–(8) do not depend on α.

In the case of three one-port standards or e.g., TMSO (thru-match-short-open) method an extension of equation set (1)–(8) without self-calibration is used. By a proper choice of equations in TMS, TMSO and some other methods all the standards do not have to be connected to both analyzer ports (cf. [20], [17]).

Some combinations of standards are singular, while some others are too error sensitive to be used in practice. For example through-short-X and through-open-X ($A_{12} = A_{21} = 1$ and $B_{11} = B_{22} = \pm 1$) or a combination of any three matched standards are all singular.

The de-embedding equations based on the eight-term error model can be written in terms of S parameters. By applying (1)–(4) for device-under-test (DUT) D instead of standard A:

$$\begin{bmatrix} L_{22}M_{D11} - \Delta L & 0 & kR_{22}M_{D12} & 0 \\ 0 & L_{22}M_{D11} - \Delta L & 0 & kR_{22}M_{D12} \\ L_{22}M_{D21} & 0 & kR_{22}M_{D22} - k\Delta R & 0 \\ 0 & L_{22}M_{D21} & 0 & kR_{22}M_{D22} - k\Delta R \end{bmatrix} \begin{bmatrix} D_{11} \\ D_{12} \\ D_{21} \\ D_{22} \end{bmatrix} = \begin{bmatrix} M_{D11} - L_{11} \\ kM_{D12} \\ M_{D21} \\ kM_{D22} - kR_{11} \end{bmatrix}. \tag{46–49}$$

Since parameters L_{ij}, R_{ij} and k are known and M_{Dij} are measured, the device S parameters $\boldsymbol{D}$ can easily be solved. For direct equations, see for example [7].

III. Application to Some Practical Methods

A. LD, "LA" and "LM" Methods

If $\boldsymbol{A}$ and $\boldsymbol{B}$ are matched (50 Ω) transmission line or attenuation network standards, $A_{11} = A_{22} = B_{11} = B_{22} = 0$. $\boldsymbol{B}$ may also be a matched load. It can be shown by direct calculation from (1)–(8) that $y_1 = x_2 = x_3 = x_4 = y_5 = y_6 = 0$ and:

$$x_1 = L_{11} = \frac{M_{A12}M_{B11} - M_{A11}M_{B12}d}{M_{A12} - M_{B12}d} \tag{50}$$

$$y_2 = \frac{L_{22}}{k} = \frac{M_{A22} - M_{B22}}{M_{A21} - M_{B21}d} * \frac{1}{A_{12}} \tag{51}$$

$$y_3 = \frac{\Delta L}{k} = \frac{\Delta M_A - M_{A11}M_{B22} + M_{A12}M_{B21}d}{M_{A21} - M_{B21}d} * \frac{1}{A_{12}} \tag{52}$$

$$y_4 = R_{11} = \frac{M_{A21}M_{B22} - M_{A22}M_{B21}d}{M_{A21} - M_{B21}d} \tag{53}$$

$$x_5 = kR_{22} = \frac{M_{A11} - M_{B11}}{M_{A12} - M_{B12}d} * \frac{1}{A_{21}} \tag{54}$$

$$x_6 = k\Delta R = \frac{\Delta M_A - M_{A22}M_{B11} + M_{A21}M_{B12}d}{M_{A12} - M_{B12}d} * \frac{1}{A_{21}} \tag{55}$$

$$d = \frac{B_{12}}{A_{12}} = \frac{B_{21}}{A_{21}} \tag{56}$$

where nonzero values of x_i and y_i are equal to the solution of the line-delay method assuming $k = 1$ [17]. In the case of line-short and line-open standards x_i and y_i are all of nonzero value. For a zero-length through $A_{12} = A_{21} = 1$.

Parameter d can be calculated from (10), [17] even if one of the standards is an attenuation network. A matched attenuator can be considered a lossy transmission line. The correct root can be chosen on the basis of the approximate line lengths or attenuation (if considerable), as in the normal TSD and LRL methods. Note that L_{11} and R_{11} become exactly correct even with only two non-reflective standards. The other four error parameters have in addition the factor $1/k$ or k, which should be exactly known to allow accurate determination of S_{11} and S_{22} of the device-under-test. When a measured DUT is de-embedded using (50)–(56), the results will be kS_{11}, S_{22}/k, S_{12} and S_{21}, as stated already in [14]. The achieved accuracy is thus dependent on the actual value of k.

B. LDX, LAX, and LMX Methods

Such solutions as LDR = LRL, LAR, LMR, LDN, LAN, and LMN are all achieved using the same formulae as in LD case, but calculating the value of k using the third standard as described in Section II. It can be shown by a straight forward solution of (30)–(33) that $u_1 = v_2 = v_3 = u_4 = 0$, provided that $A_{11} = A_{22} = B_{11} = B_{22} = 0$. The other parameters can be found as follows:

$$v_1 = \frac{y_2(M_{C11}y_4 - \Delta M_C) + y_3(M_{C22} - y_4)}{x_5(M_{C22}y_3 - \Delta M_C y_2) + x_6(M_{C11}y_2 - y_3)} \tag{57}$$

$$u_2 = \frac{x_5(M_{C22}x_1 - \Delta M_C) + x_6(M_{C11} - x_1)}{x_5(M_{C22}y_3 - \Delta M_C y_2) + x_6(M_{C11}y_2 - y_3)} \tag{58}$$

$$u_3 = \frac{M_{C21}(y_3 - x_1 y_2)}{x_5(M_{C22}y_3 - \Delta M_C y_2) + x_6(M_{C11}y_2 - y_3)} \tag{59}$$

$$v_4 = \frac{M_{C12}(x_6 - x_5 y_4)}{x_5(M_{C22} y_3 - \Delta M_C y_2) + x_6(M_{C11} y_2 - y_3)} \tag{60}$$

$$k = \pm\sqrt{\frac{u_2}{v_1}} \tag{61}$$

$$\Gamma = \pm\sqrt{v_1 u_2} \tag{62}$$

$$T = u_3 = v_4. \tag{63}$$

C. TSD Solution

A similar solution as in the TSD method [1] is achieved using (50)–(56) and (30)–(33). Both S parameters A_{12} and A_{21} and parameter k are calculated as a by-product. First d is determined from (10). Then the solutions (50)–(55) are calculated setting $A_{21} = A_{12} = 1$. These results y_2', y_3' and x_5', x_6' have to be corrected with coefficients k/A_{12} and $1/(kA_{21})$ respectively to get the correct S parameters. The coefficients and the error parameters are obtained as follows:

$$\frac{k}{A_{12}} = \frac{M_{C11} - x_1}{M_{C11} y_2' - y_3'} \frac{1}{C_{11}} \tag{64}$$

$$\frac{1}{kA_{21}} = \frac{M_{C22} - y_4}{M_{C22} x_5' - x_6'} \frac{1}{C_{22}} \tag{65}$$

$$L_{11} = x_1' = x_1 \tag{66}$$

$$L_{22} = y_2' \frac{k}{A_{12}} \tag{67}$$

$$\Delta L = y_3' \frac{k}{A_{12}} \tag{68}$$

$$R_{11} = y_4' = y_4 \tag{69}$$

$$R_{22} = x_5' \frac{1}{kA_{21}} \tag{70}$$

$$\Delta R = x_6' \frac{1}{kA_{21}}. \tag{71}$$

C_{11} and C_{22} can be any reflective (known) one-ports. Most often $C_{11} = C_{22}$. The advantage of the LDS = TSD solution is that S_{21}'s of both lines can be calculated from the measurement data, while the LRL method gives only the ratio of S_{21}'s of the lines. The reflection standard has to be exactly known in the TSD method. The solution covers also the case with an attenuation network or a matched load as standard $\boldsymbol{B}$.

D. Application of the Method of Least-Squares

If the calibration equations have redundant data, some data reduction techniques can improve the accuracy. The least-squares-fit (LSF) algorithm [15], [21] is frequently used. Matricial equation (1)–(8) and its possible extension to more standards can be written simply:

$$NE = G + kH = F. \tag{72}$$

TABLE I
VALUES OF THE SIMILARITY INDEX WITH TWO STANDARDS ASSUMING TOTAL SYMMETRY: $\delta L \cdot 10^3$, $\delta R \cdot 10^3$

Two Known Standards	With LSF	Without LSF [17]
Line, Delay	19, 19	22-27, 22-27
Line, Short	20, 19	31-44, 33-42
Line, Open	31, 31	44-46, 44-47

If parameter k is to be solved directly from (72) as in [17], the corresponding terms are first moved from matrix $\boldsymbol{F}$ to the left hand side. The pre-requisite for this is, however, at least three known standards. If matrix product $\boldsymbol{N^*N}$ is not singular, the unique solution will be

$$E = (N^*N)^{-1}N^*F \tag{73}$$

where superscript * means the complex conjugate of the transpose.

With a transmission type standard as $\boldsymbol{A}$ and a one-port standard as $\boldsymbol{B}$ there are no redundant equations, if total symmetry is not assumed. So, there is no need for data reduction. Also with two two-port standards the effect of the LSF algorithm is minimal. Under the assumption of total symmetry, however, significant decrease of error is encountered.

To test the effect of the least squares algorithm, an APLAC [22] simulation with the same network as in [17] was performed assuming total symmetry. See details in [17]. The results of the error networks $\boldsymbol{L}$ and $\boldsymbol{R}$ are shown in Table I.[1]

In the simulation of a symmetrical test fixture with two-tier calibration, two fixture standards with LSF seem to give equally accurate results as three standards. Future work will show, if this is true also in practical measurements.

IV. CONCLUSION

A novel algorithm is presented, that can handle nearly all the known calibration procedures for network analyzers and test fixtures. When used with two partly known matched standards in addition to an unknown third standard, the algorithm leads to rather compact equations. In its general form the algorithm is based on two sets of linear equations. There are no general restrictions on the S parameters of the standards. If redundant data are available due to extra calibration standards or assumption of symmetrical test fixtures, the least-squares-fit algorithm is recommended.

ACKNOWLEDGMENT

The author wishes to thank Professor M. Valtonen, T. Veijola, and R. Karhu for their kind help. M. Sipilä, K. Lehtinen and T. Malmivaara in addition to the staff of the

[1]In [17] the Line standard was named "Thru."

Circuit Theory Laboratory are thanked for many useful discussions.

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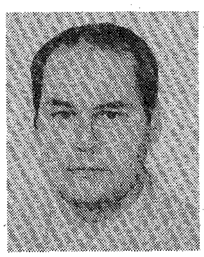

Kimmo J. Silvonen was born in Koski Hl, Finland on October 10, 1957. He received the degrees of Dipl.Eng. (M.Sc.) and Lic.Tech. in electrical engineering from the Helsinki University of Technology in 1983 and 1988, respectively.

Since 1979 he has been employed by the Circuit Theory Laboratory at the Helsinki University of Technology, first as a Student Assistant and thereafter as a Teaching and Research Assistant, and a Lecturer. During the academic year 1990–1991 he was an Acting Professor of Circuit Theory at the institution. Currently, he continues as a Lecturer and is performing research towards the Ph.D. degree. His current research interests are network analyzer measurements and microwave circuit theory including microstrip circuits.